Analysis of CDMA Signal Spectral Regrowth and Waveform Quality

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***Abstract* — Spectral regrowth and waveform quality of a reverse-link CDMA signal passed through a weakly non-linear circuit are analyzed using the power series and statistical methods. The analysis is based on a time-domain model of the signal that gives more accurate distortion estimates than the widely used Gaussian noise assumption. The model is also used to derive the probability density function of the CDMA signal to show its difference from that of the Gaussian noise.**

I. Introduction

Due to the time-varying CDMA signal envelope, the 3rd-order distortion has a significant consideration in IS-95 systems. The cross modulation of a single-tone jammer dictates the linearity requirement of the receiver front end. The spectral regrowth and waveform quality of the CDMA signal are the key parameters of the transmitter. The cross modulation and spectral regrowth have already been analyzed in the literature [1]-[3]. But the effect of the 3rd-order distortion on the CDMA waveform quality hasn't received a wide attention yet.

Predicting distortion in a CDMA system has always been challenging due to the random nature of the CDMA signal. The latter has often been modeled as a narrow-band Gaussian noise (NBGN) [2], [3] to simplify statistical analysis by using the known expansions of higher-order normal moments. [1] showed that the NBGN assumption leads to inaccurate estimates of the cross-modulation distortion in CDMA receivers and proposed a more accurate model of the reverse-link CDMA signal based on its description in the time domain.

This model is used here to find the probability density function (PDF) of the CDMA signal and compare it to the Gaussian PDF. The spectral regrowth and waveform quality of the CDMA signal passed through a weakly nonlinear circuit are analyzed using the power series and statistical methods. The analytical results are compared to the measured data.

II. Time-Domain Model and PDF of CDMA Signal

The time-domain model of a CDMA signal proposed in [1] is derived according to the IS-95 CDMA reverse-link modulation scheme shown in Fig. 1. The transmitted Walsh-coded data is first split into the I and Q channels and spread by the orthogonal PN codes at the rate B of 1.2288Mcps. The Q-channel data is delayed by ½ chip resulting in OQPSK spreading (T in Fig. 1 is the chip time equal to $1/B$). The spread data streams can be approximated with impulse trains as

$$I_{data}(t) = \sum_{k=-\infty}^{\infty} i_k \delta(Bt + \phi/\pi - k)$$

$$Q_{data}(t) = \sum_{k=-\infty}^{\infty} q_k \delta(Bt + \phi/\pi - k + 1/2)$$

where $\delta(t)$ is the Dirac delta function, ϕ is a random phase uniformly distributed in $(0, \pi)$ and i_k and q_k are independent random numbers taking values of ± 1 with equal probability. These binary numbers are the results of the modulo-2 addition of the data bits with the spreading code chips. Due to the PN code randomness, the various numbers within the same spread-data stream are also considered independent.

With an acceptable accuracy, the IS-95 baseband pulse-shaping filter can be modeled as an ideal low-pass filter with the cutoff frequency $B/2$ and impulse response $h(t)= B\cdot\text{sinc}(Bt)$ where $\text{sinc}(z)=\sin(\pi z)/(\pi z)$. The I and Q spread-data signals passed through the filter become

$$I(t) = \sum_{k=-\infty}^{\infty} i_k \text{sinc}(Bt + \phi/\pi - k)$$

$$Q(t) = \sum_{k=-\infty}^{\infty} q_k \text{sinc}(Bt + \phi/\pi - k + 1/2)$$

The filtered I and Q signals are modulated on two carriers in quadrature and summed producing the transmitted signal

$$c(t) = i(t) + q(t) \qquad (1)$$

where $i(t)=I(t)\cos(\omega_c t+\theta)$, $q(t)=Q(t)\sin(\omega_c t+\theta)$, ω_c is the angular frequency of the carriers and θ is their random phase independent of ϕ and uniformly distributed in $(0, 2\pi)$. Thanks to ϕ and θ, the CDMA signal and its distortion can be treated as stationary processes. Equation (1) is the time-domain model of the unity-variance CDMA signal.

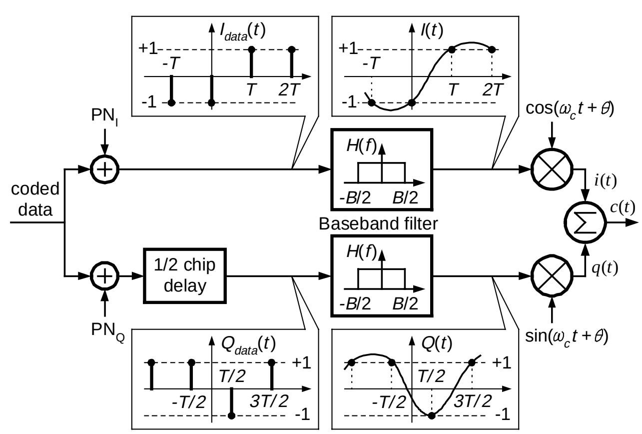

Fig.1. Simplified block diagram of IS-95 CDMA reverse-link modulator.

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By definition, the characteristic function of $c(t)$ is

$$M(v) = E\{\exp[jvc(t)]\} = E\{\exp[jvi(t)]\exp[jvq(t)]\}$$
$$= \frac{1}{2\pi^2}\int_0^{2\pi} d\theta \int_0^{\pi} d\phi \; E_{i_k}\{\exp[jvi(t)]\} \; E_{q_k}\{\exp[jvq(t)]\}$$

where $E\{\}$ is the statistical average and $E_{i_k}\{\}$ and $E_{q_k}\{\}$ are the averages over i_k and q_k respectively. Since $c(t)$ is stationary, t can be set to 0 without affecting the result.

$$E_{i_k}\{\exp[jvi(0)]\} = E_{i_k}\left\{\exp\left[jv\cos(\theta)\sum_{k=-\infty}^{\infty} i_k \mathrm{sinc}(\phi/\pi - k)\right]\right\}$$
$$= \prod_{k=-\infty}^{\infty} E_{i_k}\{\exp[jv\cos(\theta)\; i_k \mathrm{sinc}(\phi/\pi - k)]\}$$
$$= \prod_{k=-\infty}^{\infty} \left\{ \begin{matrix} 0.5\exp[jv\cos(\theta)(+1)\mathrm{sinc}(\phi/\pi - k)] \\ +\,0.5\exp[jv\cos(\theta)(-1)\mathrm{sinc}(\phi/\pi - k)] \end{matrix} \right\}$$
$$= \prod_{k=-\infty}^{\infty} \cos[v\cos(\theta)\;\mathrm{sinc}(\phi/\pi - k)].$$
$$E_{q_k}\{\exp[jvq(0)]\} = \prod_{k=-\infty}^{\infty} \cos[v\sin(\theta)\;\mathrm{sinc}(\phi/\pi - k + 1/2)].$$

So, the characteristic function of the CDMA signal is

$$M(v) = \frac{1}{2\pi^2}\int_0^{2\pi} d\theta \int_0^{\pi} d\phi \prod_{k=-\infty}^{\infty} \cos\left[v\,\cos(\theta)\,\frac{\sin(\phi)}{\phi - k\pi}\right] \cdot \cos\left[v\sin(\theta)\frac{\cos(\phi)}{\phi - k\pi + \pi/2}\right].$$

The above integral belongs to a group of sinc-related integrals that don't have analytical solutions according to [4]. It was computed numerically here and then converted to PDF using the inverse Fourier transform and plotted in Fig. 2 together with PDF of NBGN. This graph shows that PDF of the CDMA signal significantly differs from that of NBGN.

III. Analysis of CDMA Signal Distortion

Let the output signal of a nonlinear circuit be expanded in a power series in terms of the input signal $x(t)$ as

$$y(t) = y_1(t) + y_2(t) + y_3(t) + \ldots \;\text{ with }\; y_n(t) = a_n x^n(t)$$

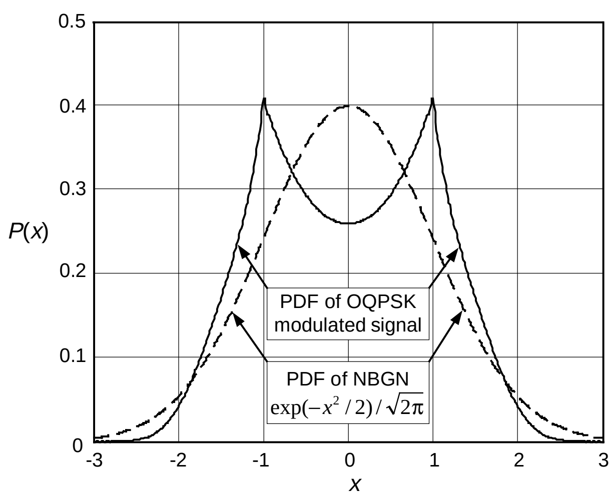

Fig. 2. Probability density functions of CDMA signal with OQPSK modulation and narrow-band Gaussian noise.

where $x(t) = \overline{V_c}c(t)$, $\overline{V_c^2}$ is the CDMA signal mean-square voltage (or variance) and a_n's are the expansion coefficients. Terms of the order higher than three will be neglected here i.e. the circuit is assumed *weakly nonlinear*. $y_2(t)$ will be omitted as well since it doesn't contribute to the in-band distortion.

The autocorrelation function of the output is then

$$R_y(\tau) = E\{[y_1(0) + y_3(0)]\cdot[y_1(\tau) + y_3(\tau)]\}$$
$$= E\{y_1(0)y_1(\tau)\} + E\{y_1(0)y_3(\tau)\} + E\{y_3(0)y_1(\tau)\} + E\{y_3(0)y_3(\tau)\} \quad (2)$$

The first summand in (2) is the autocorrelation of the linear response of the circuit $y_1(t)$. The 2nd and 3rd summands are the crosscorrelations describing the in-channel portion of the 3rd-order response $y_3(t)$ correlated to $y_1(t)$ and responsible for the gain compression or expansion. The last summand is the autocorrelation of $y_3(t)$ that gives the spectral regrowth.

A. Linear Response

The autocorrelation function of the linear response is

$$E\{y_1(0)y_1(\tau)\} = a_1^2\overline{V_c^2}E\{c(0)\,c(\tau)\} = P_o\,\mathrm{sinc}(B\tau)\cos(\omega_c\tau)$$

where $P_o = a_1^2\overline{V_c^2}$ is the total output power of the linear response. The Fourier transform of the above expression gives the following power spectral density (PSD) function

$$S_{lin}(\Omega) = P_o/(2B) \quad \text{for} \quad |\Omega| \le 1$$

where $\Omega = 2(|f| - f_c)/B$ is the normalized frequency offset.

The derived PSD is easier to use if it is converted to the power in a frequency range $\Delta f << B$. Δf corresponds to the resolution bandwidth of a spectrum analyzer typically selected to be 30kHz for ACPR measurements. Integrating $S_{lin}(\Omega)$ over Δf and taking into account the negative frequency spectrum (adds factor of 2) result in

$$P_{lin}(\Omega) = P_o\Delta f / B \quad \text{for} \quad |\Omega| \le 1.$$

B. Distortion Causing Gain Compression or Expansion

The 2nd summand in (2) can be evaluated as follows

$$R_{y_1y_3}(\tau) = E\{y_1(0)\;y_3(\tau)\} = a_1a_3\overline{V_c^4}E\{(i+q)(i_\tau + q_\tau)^3\}$$
$$= a_1a_3\overline{V_c^4}E\{ii_\tau^3 + qq_\tau^3 + 3ii_\tau q_\tau^2 + 3i_\tau^2 qq_\tau\} \quad (3)$$

where i, q, i_τ and q_τ denote $i(0)$, $q(0)$, $i(\tau)$ and $q(\tau)$, respectively. The products iq_τ^3, $i_\tau^3 q$, $3ii_\tau^2 q_\tau$, and $3i_\tau qq_\tau^2$ are omitted because their averages are zero.

Let g_i denote $\mathrm{sinc}(\phi/\pi - k)$, h_i denote $\mathrm{sinc}(B\tau + \phi/\pi - k)$, g_q denote $\mathrm{sinc}(\phi/\pi - k + ½)$, h_q denote $\mathrm{sinc}(B\tau + \phi/\pi - k + ½)$, and parentheses denote summation over k so that

$$(g_i^m) = \sum_{k=-\infty}^{\infty}\mathrm{sinc}^m(\phi/\pi - k) \quad \text{and}$$
$$(g_i^m h_i^n) = \sum_{k=-\infty}^{\infty}\mathrm{sinc}^m(\phi/\pi - k)\;\mathrm{sinc}^n(B\tau + \phi/\pi - k). \quad (4)$$

Then each summand in (3) can be evaluated in terms of the sums similar to (4). For example,

$$E\{ii_\tau^3\} = E_\theta\{\cos(\theta)\cos^3(\omega_c\tau + \theta)\}\,E\{I(0)\,I^3(\tau)\}$$
$$= 3/8\cos(\omega_c\tau)\;E\{I(0)\,I^3(\tau)\}$$

where $E\{I(0)I^3(\tau)\}$ is the 4th-order joint moment of $I(t)$. Using the joint moment expansions for a binary pulse train [5], we get

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$$E\{I(0)\,I^3(\tau)\} = E_\phi\{3(g_i h_i)(h_i^2) - 2(g_i h_i^3)\}\ .$$

The first two sums in this expansion are well known: $(g_i h_i)$= sinc($B\tau$), $(h_i^2)=1$ [6]. Other sums can be found using Mathcad.

Substituting all evaluated summands into (3), we get

$$R_{y_1y_3}(\tau) = \frac{3a_1a_3\overline{V_c^4}}{4}\cos(\omega_c\tau)\left[3\,\mathrm{sinc}(B\tau) - \frac{\mathrm{sinc}(B\tau)-\cos(\pi B\tau)}{(\pi B\tau)^2}\right]$$

The 3rd summand in (2) is equal to the 2nd one. The Fourier transform of $2R_{y_1y_3}(\tau)$ gives the following PSD of the in-channel distortion

$$S_{ICD}(\Omega) = 3a_1a_3\overline{V_c^4}(\Omega^2+5)/(8B) \quad \text{for} \quad |\Omega|\le 1\ . \qquad (5)$$

If the sign of a_3 is opposite to that of a_1, S_{ICD} is subtracted from S_{lin} causing the gain compression. If the signs are the same, S_{ICD} is added to S_{lin} resulting in the gain expansion. In the following analysis, the gain compression is assumed.

The 3rd-order expansion coefficient a_3 can be expressed as a function of the output 3rd-order intercept point OIP_3 as

$$a_3 = -2a_1^3/(3\cdot OIP_3)\ .$$

Substituting a_3 into (5) and converting PSD into the power in Δf, we get

$$P_{ICD}(\Omega) \approx -P_o^2\Delta f(\Omega^2+5)/(2B\cdot OIP_3) \quad \text{for} \quad |\Omega|\le 1\ .$$

C. Spectral Regrowth

The autocorrelation function of the 3rd-order response is

$$E\{y_3(0)\,y_3(\tau)\} = a_3^2\overline{V_c^6}E\{i^3i_\tau^3 + q^3q_\tau^3 + 3i^3i_\tau q_\tau^2 + 3i^2qq_\tau^3 + 3ii_\tau^3q^2 + 3i_\tau^2q^3q_\tau + 9i^2i_\tau^2qq_\tau + 9ii_\tau q^2q_\tau^2\}.$$

Lengthy derivations give the following PSD function of the 3rd-order distortion centered at ω_c

$$S_{SR}(\Omega) = \begin{cases} \dfrac{3a_3^2\overline{V_c^6}}{32B}[3\Omega^2(\Omega^2+1)+22], & |\Omega|\le 1 \\ \dfrac{3a_3^2\overline{V_c^6}}{128B}(3-|\Omega|)^2(3\Omega^2-8|\Omega|+9), & 1<|\Omega|\le 3. \end{cases}$$

Converting $S_{SR}(\Omega)$ into the power within Δf gives

$$P_{SR}(\Omega) \approx \begin{cases} \dfrac{P_o^3\Delta f[3\Omega^2(\Omega^2+1)+22]}{12\cdot OIP_3^2B}, & |\Omega|\le 1 \\ \dfrac{P_o^3\Delta f(3-|\Omega|)^2(3\Omega^2-8|\Omega|+9)}{48\cdot OIP_3^2B}, & 1<|\Omega|\le 3. \end{cases}$$

The power of the combined response centered at ω_c is

$$P_{comb}(\Omega) = P_{lin}(\Omega) + P_{ICD}(\Omega) + P_{SR}(\Omega)\ .$$

Output spectra of the CDMA signal and NBGN are plotted in Fig. 3 for P_o=-3.9dBm, OIP_3=+12.2dBm and Δf=30kHz. NBGN exhibits 5.1dB more spectral regrowth at 885kHz offset and 3.7dB more at 1.25MHz offset than the CDMA signal.

IV. Waveform Quality Factor

Waveform quality factor ρ is a fraction of energy of the actual CDMA signal within the channel that correlates with the ideal reference signal. If the transmitted signal matches the reference perfectly, ρ=1 i.e. 100% of the signal is useful information to the receiver. If ρ<1, the uncorrelated signal power appears as an added noise interfering with other users in the cell and reducing its capacity. IS-95 sets the minimum ρ at 0.944 for mobile transmitters. ρ is reduced by the transmitter impairments such as the PN code time offset, magnitude, phase and frequency errors in the I/Q modulator, the carrier phase noise and feedthrough, the in-channel thermal noise and the 3rd-order distortion [7]. Here only the distortion effect will be considered.

Since the linear response $y_1(t)$ is the scaled ideal waveform, ρ can be computed as the *power correlation coefficient* (defined as the squared correlation coefficient) between $y_1(t)$ and the in-channel portion of the combined response $y(t)$. With the subscript '*IC*' denoting the in-channel response,

$$\rho = \frac{E\{y_1(0)y_{IC}(0)\}^2}{E\{y_1^2(0)\}E\{y_{IC}^2(0)\}}$$

where $E\{y_1(0)y_{IC}(0)\}^2/E\{y_1^2(0)\}$ is the power of the ideal signal in $y(t)$ and $E\{y_{IC}^2(0)\}$ is the total in-channel power. Substituting $y_{IC}(0) = y_1(0)+ y_{3IC}(0)$, we get

$$\rho = \frac{[E\{y_1^2(0)\}+E\{y_1(0)y_{3IC}(0)\}]^2}{E\{y_1^2(0)\}[E\{y_1^2(0)\}+2E\{y_1(0)y_{3IC}(0)\}+E\{y_{3IC}^2(0)\}]}\ .$$

It can be shown that $E\{y_1(0)\ y_{3IC}(0)\}$=$E\{y_1(0)\ y_3(0)\}$= $R_{y_1y_3}(0)$ and $E\{y_{3IC}^2(0)\}$ is the total in-channel power of $y_3(t)$. So,

$$\rho = \frac{[1-4P_o/(3\cdot OIP_3)]^2}{1-8P_o/(3\cdot OIP_3)+59P_o^2/(30\cdot OIP_3^2)}\ .$$

According to the above analysis, $y_3(t)$ contains a term with the power $2E\{y_1(0)y_3(0)\} = \pm 8P_o^2/(3\cdot OIP_3)$ correlated to the linear response and causing either gain compression (-) or expansion (+) depending on the sign of a_3. The remainder has the power $E\{y_3^2(0)\}$ and also contains a term correlated to $y_1(t)$ that causes the gain expansion regardless of a_3 sign. This term has the power $E\{y_1(0)y_3(0)\}^2/E\{y_1^2(0)\}=\ 16P_o^3/(9\cdot OIP_3^2)$. The remaining in-channel portion of $y_3(t)$ has the power $17P_o^3/(90\cdot OIP_3^2)$ and acts as the added noise.

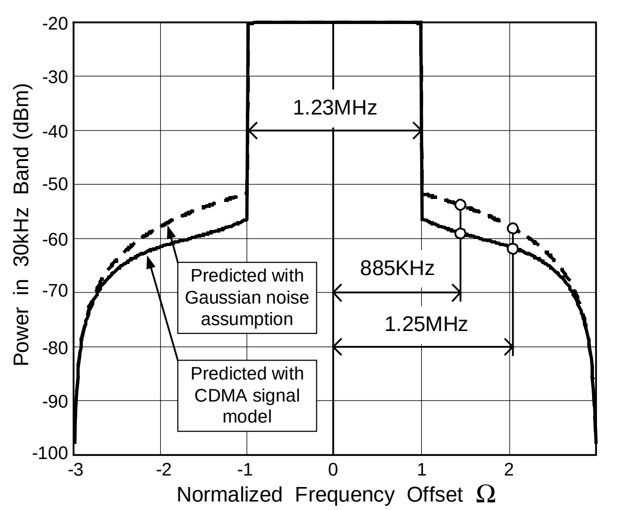

Fig. 3. Output spectra of weakly nonlinear circuit predicted with CDMA signal model and Gaussian noise assumption.

V. MEASURED RESULTS

Fig. 4 shows measured and predicted output spectra of an amplifier driven by a CDMA signal with OQPSK modulation. The total in-channel output power is –4.0 dBm. The corresponding linear power P_o extrapolated from the small-signal measurements is –3.9dBm. The measured OIP_3 is +12.2dBm. The predicted spectrum closely follows the measured one at most frequencies. The disagreement at the channel edges is due to a non-ideal roll off of the baseband filter response at the cut off frequency.

Fig. 5 shows the dependence of the output power in the channel and at 1.25MHz offset on the input power. Fig. 6 shows the waveform quality factor as a function of the input power. All powers are measured in 30kHz band. The disagreement between the measured and predicted results at higher input levels is explained by a stronger distortion contribution of the 5th and higher odd-degree nonlinearities of the amplifier that have been neglected in the analysis. The deviation of the measured ρ from the predicted one at low input levels is caused by the clock jitter and other impairments of the test setup.

VI. CONCLUSION

Using the time-domain model of the reverse-link CDMA signal, its PDF is shown to be different from that of a narrow-band Gaussian noise. This difference makes the NBGN model unsuitable for analyzing the distortion in IS-95 CDMA systems. The time-domain model gives more accurate distortion estimates. The derived closed-form expressions for the spectral regrowth and waveform quality factor can be used in system design and analysis. But their application is limited to the cases where the 3rd-order nonlinearity is dominant.

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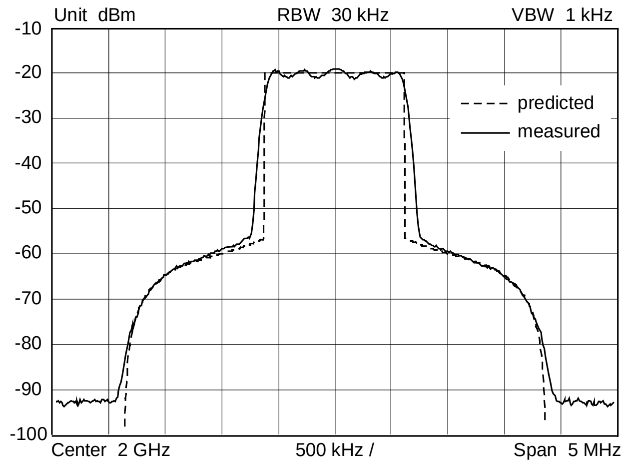

Fig. 4. Measured and predicted output spectra of amplifier.

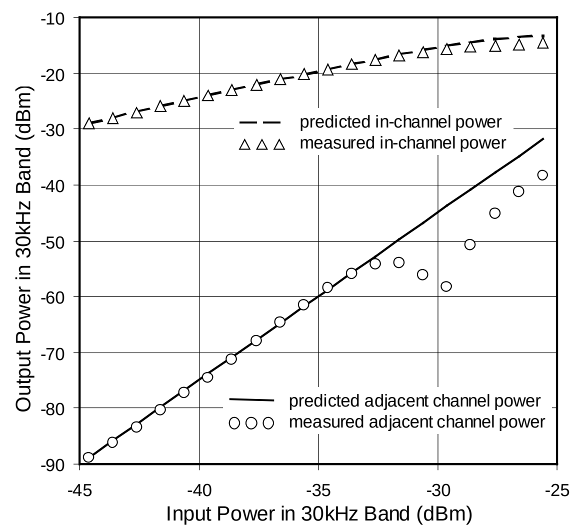

Fig. 5. In-channel and adjacent channel powers vs. input power.

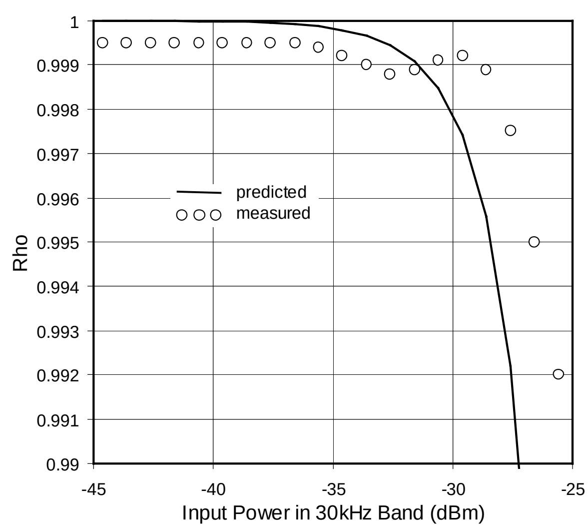

Fig. 6. Waveform quality factor ρ vs. input power.